

# Electroweak corrections to $t\bar{t}$ production at hadron colliders

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## Abstract

We report on our recent work on electroweak corrections to  $t\bar{t}$  production at hadron colliders. Specifically, we discuss the weak-interaction contributions to the top quark transverse momentum and  $t\bar{t}$  invariant mass distributions and an induced parity-violating top-spin asymmetry.

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LHC is planned to start operation within this year. Once LHC will run near design luminosity, huge number of  $t\bar{t}$  events will be produced. This will allow detailed exploration of the properties of top quarks. For this aim, theoretical predictions related to top quark production and decay should be made as precisely as possible within the SM. Predictions for unpolarized  $t\bar{t}$  production have long been known at next-to-leading order (NLO) QCD [1], and these NLO results were refined by resummation of soft gluon and threshold logarithms [2]. Moreover,  $t\bar{t}$  production and decay including the full spin degrees of freedom of the intermediate  $t$  and  $\bar{t}$  resonances were determined to NLO QCD some time ago [3, 4]. Spin effects of top quarks can be reliably predicted in view of the short-distance nature of their interactions, and are expected to play an important role in refined data analysis.

A complete NLO analysis of  $t\bar{t}$  production within the SM should include also the electroweak radiative corrections. Though they turn out to be marginal for the production cross section  $\sigma_{t\bar{t}}$  at the Tevatron and at the LHC, they may be important for distributions at large transverse top-quark momentum or large  $t\bar{t}$  invariant mass, due to large Sudakov logarithms [5]. Moreover, the weak interactions induce small parity-violating effects, and for full exploration and interpretation of future data it is important to obtain definite SM predictions also for these effects. Weak interaction corrections to hadronic  $t\bar{t}$  production have been studied in a number of papers. The order  $\alpha_s^2\alpha$  weak-QCD corrections to  $q\bar{q} \rightarrow t\bar{t}$  and  $gg \rightarrow t\bar{t}$  were analyzed in [6] (c.f. also [7]). For  $q\bar{q} \rightarrow t\bar{t}(g)$ , full determinations of these corrections, including the infrared-divergent box contributions and the corresponding real gluon radiation, were made in [8, 9]. The order  $\alpha_s^2\alpha$  corrections to  $gg \rightarrow t\bar{t}$  including the quark triangle diagrams  $gg \rightarrow Z \rightarrow t\bar{t}$  were investigated in [10, 12, 13], and additional contributions in [11]. The purely photonic corrections were recently also calculated [14]. Parity violation in  $t\bar{t}$  production was analyzed within the SM in [7, 8, 10, 11, 15, 16, 17]. Investigations of non-SM effects include [16, 18].

At hadron colliders top quark pairs are produced predominantly by the strong interactions. The QCD corrections for the subprocesses  $i \rightarrow t\bar{t} + X$ , ( $i = q\bar{q}, gg, gq, g\bar{q}$ ) are known to order  $\alpha_s^3$ . As mentioned above, the leading corrections involving electroweak interactions are also available. We shall focus here on weak-interaction contributions, but shall mention for completeness also QED corrections. For the partonic process

$$g + g \rightarrow t + \bar{t}, \quad (1)$$

the leading electroweak contributions are of order  $\alpha\alpha_s^2$ , while for

$$q + \bar{q} \rightarrow t + \bar{t}, \quad q \neq b, \quad (2)$$

there are the order  $\alpha^2$  Born contributions (from  $q\bar{q} \rightarrow \gamma, Z \rightarrow t\bar{t}$ ) and the mixed QCD electroweak corrections of order  $\alpha_s^2\alpha$ . Due to color conservation there are no order  $\alpha_s\alpha$  interference terms for the s-channel amplitudes. But for

$$b\bar{b} \rightarrow t\bar{t} \quad (3)$$

there is, at leading order, a t-channel  $W$  exchange contribution together with the s-channel gluon,  $Z$  and  $\gamma$  exchange amplitudes. The t-channel term interferes with the QCD Born amplitude. Thus the leading electroweak contributions to (3) are of order  $\alpha^2$  and  $\alpha\alpha_s$ . In view of the large parton luminosity for  $qg$  scattering at the LHC, one should take into account also the reactions

$$gq \rightarrow t\bar{t}q, \quad g\bar{q} \rightarrow t\bar{t}\bar{q} \quad (4)$$

and determine the weak corrections to the respective amplitudes. Thus the NLO weak corrections to hadronic  $t\bar{t}$  production can be divided into three parts: i) the contributions of order  $\alpha^2$  and  $\alpha\alpha_s$  to the process (3), ii) the contributions of order  $\alpha^2\alpha_s$  and  $\alpha\alpha_s^2$  to the processes (4), and iii) the contributions of order  $\alpha^2$  and  $\alpha\alpha_s^2$  to the reactions (1) and (2). In our analysis, we employ the 5-flavor scheme [19], where the (anti)proton is considered to contain also  $b$  and  $\bar{b}$  quarks in its partonic sea. Thus the reaction (3) is a leading-order (LO) process in this scheme, while (4), for  $q = b$ , is a next-to-leading order correction to (3). As to the pure QED corrections which were calculated in [14]: it was pointed out in that work that the dominant photonic corrections are due to photon-gluon fusion,  $\gamma + g \rightarrow t\bar{t}$ .

For  $t\bar{t}$  production at the level of hadronic collisions, the inclusive spin-summed  $t\bar{t}$  cross section may be written, to NLO in the SM couplings, in the form  $\sigma = \sigma^{(0)} + \delta\sigma^{(1)} + \delta\sigma^W + \delta\sigma^{QED}$ , where the first and second term are the LO (order  $\alpha_s^2$ ) and NLO (order  $\alpha_s^3$ ) QCD contributions, while the third and fourth term result from the electroweak corrections to the processes discussed above. Table 1 contains the contributions from  $gg \rightarrow t\bar{t}(g)$  and  $q\bar{q} \rightarrow t\bar{t}(g)$  at NLO QCD<sup>1</sup>, the weak corrections of order  $\alpha_s^2\alpha$  and  $\alpha^2$ , and the QED corrections of order  $\alpha_s^2\alpha$  and  $\alpha^2\alpha_s$ . The numbers for “weak” in this table do not contain the contribution from

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<sup>1</sup>In this table, we have omitted the order  $\alpha_s^3$  contributions from  $qg$  and  $\bar{q}g$  initial states, as this table serves only the purpose of exhibiting the relative size of the electroweak corrections.

Table 1: The  $t\bar{t}$  cross section at the Tevatron ( $\sqrt{s} = 1.96\text{TeV}$ ) and at the LHC ( $\sqrt{s} = 14\text{TeV}$ ) in units of pb, for  $m_t = 172.7\text{ GeV}$ ,  $m_H = 120\text{ GeV}$  and three different values of  $\mu$ . We put  $\mu = \mu_R = \mu_F$ . The numbers for the QCD and weak contributions were obtained using the NLO parton distribution functions CTEQ6.1M, while the NLO QED contributions are from [14] which used the PDF set [21].

		$\mu = m_t/2$	$\mu = m_t$	$\mu = 2m_t$
Tevatron (pb)	NLO QCD	7.493	7.105	6.314
	Weak / QED	0.0339	0.0355	0.0346 / -0.102
LHC (pb)	NLO QCD	868.150	850.385	793.543
	Weak / QED	-14.127	-10.790	-8.368 / 4.78

the t-channel  $W$ -exchange in (3). For the evaluation of the QCD and weak contributions we have used  $\overline{\text{MS}}$  factorization and the NLO parton distribution functions (PDF) CTEQ6.1M [20]. The QED corrections are from [14], where DIS factorization and the PDF set from [21] was used, which contain PDFs at NLO QCD and NLO QED. The table shows that the weak correction to the total cross section is negative at the LHC and amounts to about  $-1.3\%$ , while it is about  $0.5\%$  at the Tevatron. The pure NLO QED correction is about  $0.6\%$  ( $-2\%$ ) at the LHC (Tevatron). Thus, the electroweak corrections to the cross section are much smaller than the scale uncertainties of the fixed-order NLO QCD corrections.

However, as we shall show below, for a number of distributions, which are among the key observables in the tool-kit for the search of new physics in  $t\bar{t}$  events, these corrections do matter if one aims at predictions with a precision at the percent level.

For the computation of these distributions we use  $m_t = 172.7\text{ GeV}$ ,  $\alpha_s(2m_t) = 0.1$ , and  $\alpha(2m_t) = 1/126.3$ . The LO QCD terms and the contributions of the corrections i) and iii) to the distributions are evaluated with the LO parton distribution functions (PDF) CTEQ6.L1, while for the computation of the contributions from ii), which depend on the factorization scale, the set CTEQ6.1M [20] is used. The scale  $\mu_F$  is varied between  $m_t/2 \leq \mu_F \leq 2m_t$ . Dependence on the renormalization scale  $\mu_R$  enters only via the  $\overline{\text{MS}}$  coupling  $\alpha_s$ . The ratio of the corrections iii) and  $d\sigma_{LO}$  is practically independent of  $\alpha_s$ , while the corresponding ratios involving i) and ii) vary weakly with  $\mu_R$ .

Fig. 1a shows the various weak-interaction contributions to the transverse mo-

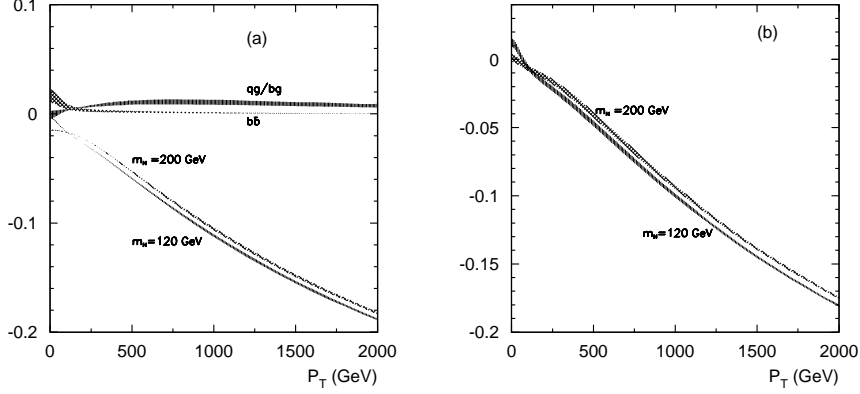


Figure 1: a) Ratios  $(d\sigma_{weak}/dp_T)/(d\sigma_{LO}/dp_T)$ , where  $d\sigma_{weak}$  are the weak-interaction corrections i), ii), and iii) to the reactions (3), (4), and  $q\bar{q}, gg \rightarrow t\bar{t}$  ( $q \neq b$ ), respectively. The latter corrections are shown for two different values of the Higgs boson mass. The hatched areas arise from scale variations as described in the text. b) Sum of the ratios shown in a) for two different values of  $m_H$ .

momentum distribution of the top quark at the LHC, normalized to  $d\sigma_{LO}/dp_T$ . The hatched areas depict the range of values when  $\mu \equiv \mu_F = \mu_R$  is varied between  $m_t/2$  and  $2m_t$ . Fig. 1a shows that the weak correction i) to the  $p_T$  distribution of the top quark is positive and small. Its significance is confined to the region  $p_T \leq 100$  GeV, where it dominates the other weak corrections. However, in this region these corrections make up only between 1% and 2% of the LO QCD  $p_T$  distribution. In the high  $p_T$  region, where the weak-interaction corrections to the  $p_T$  spectrum become larger, the contributions from the processes (3) and (4) become less significant in comparison to the weak corrections iii). Fig. 1b displays the ratio of the sum of the weak corrections i), ii), and iii) and the LO QCD contribution. The corrections are negative in almost the whole  $p_T$  range. For large  $p_T$  they are quite sizeable; for instance, for  $p_T = 1000$  GeV they amount to  $-10\%$  of the LO QCD contribution. The photonic corrections to the  $p_T$  spectrum are also negative, but smaller in magnitude [14]. For instance, at  $p_T = 1000$  GeV they amount to  $-2\%$  of  $d\sigma_{LO}/dp_T$ .

In Fig. 2a the analogous ratios are displayed for the  $M_{t\bar{t}}$  distribution. The weak-interaction corrections i) and ii) are both positive and show a considerable scale uncertainty. They reduce the magnitude of the leading weak corrections iii),

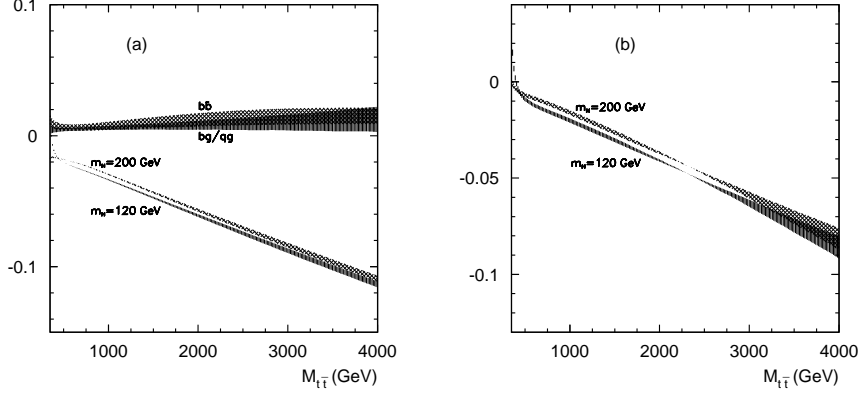


Figure 2: a) Ratios  $(d\sigma_{weak}/dM_{t\bar{t}})/(d\sigma_{LO}/dM_{t\bar{t}})$ , where  $d\sigma_{weak}$  refers to the weak-interaction corrections i), ii), and iii). The latter corrections are shown for two different values of the Higgs boson mass. The hatched areas arise from scale variations as described in the text. b) Sum of the ratios shown in a) for two different values of  $m_H$ .

which are negative, as shown in Fig. 2b. Notice that in these ratios the changes of  $d\sigma_{weak}$  and  $d\sigma_{LO}$  due to variations of the LO PDF and the LO QCD coupling with  $\mu$  cancel to a large extent. How large are the photonic corrections? The authors of [14] have not computed the  $M_{t\bar{t}}$  distribution, but the distribution of the partonic c.m. energy, and obtained that the QED correction to this quantity is quite small ( $\leq 1\%$  in magnitude) in most of the kinematic range.

Weak interaction-corrections can induce also parity-violating (PV) effects. One possibility to search for PV in hadronic  $t\bar{t}$  pair production is to check whether the produced ensemble of  $t$  and  $\bar{t}$  quarks is longitudinally polarized. Let us discuss this possibility in the more general context of top-spin physics in  $t\bar{t}$  pair production and decay. The polarizations and spin-spin correlations which is imprinted upon the  $t\bar{t}$  sample by the specific production dynamics lead, through the parity-violating weak decays of these quarks, to characteristic angular distributions and correlations among the final state particles/jets. In semileptonic top-quark decays the outgoing charged lepton is, according to the SM, the best top-spin analyzer, while for non-leptonic top decays the resulting least-energetic non- $b$  jet is a good and experimentally acceptable choice [22]. Of the main  $t\bar{t}$  decay modes, that is, the all-jets, lepton + jets, and dilepton channels, very probably only the latter two

are useful for top-spin physics, because the all-jets channels have rather low top-spin analyzer quality and large backgrounds. Thus, for measuring top-spin effects in  $t\bar{t}$  production and decay at the Tevatron or LHC one may consider the reactions

$$p\bar{p}, pp \rightarrow t\bar{t} + X \rightarrow a(\mathbf{p}_+) + \bar{b}(\mathbf{p}_-) + X, \quad (5)$$

where  $a$  and  $\bar{b}$  denotes either a charged lepton ( $\ell = e, \mu$ ) or a jet from  $t$  and  $\bar{t}$  decay, respectively, and  $\mathbf{p}_+$  and  $\mathbf{p}_-$  denote the 3-momenta of these particles/jets in the respective  $t$  and  $\bar{t}$  rest frame<sup>2</sup>. One may now choose two polar vectors  $\hat{\mathbf{a}}$  and  $\hat{\mathbf{b}}$  as reference axes, determine the angles  $\theta_+ = \angle(\mathbf{p}_+, \hat{\mathbf{a}})$  and  $\theta_- = \angle(\mathbf{p}_-, \hat{\mathbf{b}})$  event by event, and consider the double distribution

$$\frac{1}{\sigma_{ab}} \frac{d\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1}{4} (1 + B_+ \cos\theta_+ + B_- \cos\theta_- - C \cos\theta_+ \cos\theta_-), \quad (6)$$

where  $\sigma_{ab}$  is the cross section of the channel (5). The right-hand side of (6) is the a priori form of this distribution if no cuts are applied. In the presence of cuts the shape of the distribution will in general be distorted. Nevertheless, one may use the bilinear form (6) as an estimator in fits to data. The coefficient  $C$  contains the information about the parity-even  $t\bar{t}$  spin correlations. These distributions were predicted for the Tevatron and the LHC in [4] to NLO QCD for an number of reference axes. It is straightforward to add to these NLO QCD results the weak interaction corrections which may be enhanced by suitable cuts on  $M_{t\bar{t}}$ . Detailed results can be found in [8, 10].

The PV dynamics that contributes to  $t\bar{t}$  production leads to a polarization of the  $t$  and  $\bar{t}$  samples along some polar vector, i.e., to non-zero expectation values  $\langle \mathbf{S}_t \cdot \hat{\mathbf{a}} \rangle$ ,  $\langle \mathbf{S}_{\bar{t}} \cdot \hat{\mathbf{b}} \rangle$ . The information about these parity-odd (anti)top-spin effects is contained in the coefficients  $B_{\pm}$  of (6). The highest sensitivity to such effects is achieved when one uses the charged lepton from semileptonic  $t$  or  $\bar{t}$  decay as top-spin analyzer. Thus, we consider the reactions

$$p\bar{p}, pp \rightarrow t\bar{t} + X \rightarrow \ell^+(\mathbf{p}_+) + X, \quad (7)$$

where  $\ell = e, \mu$ . (Experimentally, the event selection should use  $b$ -tagging, etc. in order to discriminate against single  $t$  production, which also contributes to the final state (7).) Integrating (6) with respect to  $\cos\theta_-$  yields the distribution

$$\frac{1}{\sigma_{\ell}} \frac{d\sigma}{d\cos\theta_+} = \frac{1}{2} (1 + B_+ \cos\theta_+). \quad (8)$$

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<sup>2</sup>For the lepton + jets and for the dileptonic channels the  $t$  and  $\bar{t}$  momenta, i.e., their rest frames can be kinematically reconstructed up to ambiguities, which may be resolved with Monte Carlo methods using the matrix element of the reaction.

As to the choice of the reference axes  $\hat{\mathbf{a}}$  and  $\hat{\mathbf{b}}$ , we consider here the helicity basis [4], which is the best choice for the LHC. (For the Tevatron, a suitable choice would be the beam basis.) The distribution (8) leads to the parity-violating asymmetry

$$A_{PV} \equiv \frac{N_+ - N_-}{N_+ + N_-} = \frac{B_+}{2}, \quad (9)$$

where  $N_{\pm}$  is the number of events (7) with  $\cos \theta_+$  larger or smaller than zero.

For the computation of  $B_+$  we need several ingredients. For brevity, we consider here  $A_{PV}$  only for the LHC. Here  $t\bar{t}$  production is dominated by gluon fusion, and the top quarks are on average (moderately) relativistic. Thus, as already said above, choosing the top-quark direction of flight as the “top-spin quantization axis” approximately optimizes  $A_{PV}$  for a given PV production dynamics. At the level of the intermediate  $t\bar{t}$  events, the basic PV spin-asymmetry is the helicity asymmetry for  $t$  quarks,  $Z_{hel}$ :

$$Z_{hel} = \frac{d\sigma_+}{dM_{t\bar{t}}} - \frac{d\sigma_-}{dM_{t\bar{t}}}, \quad \text{and} \quad \Delta_{hel} = \frac{Z_{hel}}{d\sigma_{LO}/dM_{t\bar{t}}}. \quad (10)$$

The subscripts  $\pm$  in (10) refer to a  $t$  quark with positive/negative helicity while the helicity states of the  $\bar{t}$  are summed. The asymmetry coefficient  $B_+$  is given by [10]

$$B_+ = \kappa_+ \frac{\int dM_{t\bar{t}} Z_{hel}(M_{t\bar{t}})}{\sigma_{t\bar{t}}}, \quad (11)$$

where  $\kappa_+$  is the top-spin analyzing power of  $\ell^+$ . In the SM  $\kappa_+ = 1$  to lowest order and  $\kappa_+ = 0.9984$  including the order  $\alpha_s$  QCD corrections. Fig. 3a displays the weak-interaction induced contributions i), ii) and iii) to  $\Delta_{hel}$ . (As the SM Yukawa coupling is parity-conserving, iii) does not depend on  $m_H$ .) Each correction i) and ii) shows a considerable scale dependence which, however, cancels to a large extent in the sum of the two contributions – c.f. Fig. 3b. The corrections i) and ii) reduce the contribution iii) to  $\Delta_{hel}$  by about 50%. Thus we find that in the SM,  $\Delta_{hel} \leq 2\%$  for  $M_{t\bar{t}} \leq 4$  TeV. Selecting events (7) at the LHC with a  $t\bar{t}$  invariant mass larger than some minimum value, we obtain the following SM prediction for the parity-violating asymmetry  $A_{PV}$ :

$$A_{PV}(M_{t\bar{t}} > 0.5\text{TeV}) = 0.0004, \quad A_{PV}(M_{t\bar{t}} > 1\text{TeV}) = 0.0021, \quad A_{PV}(M_{t\bar{t}} > 1.5\text{TeV}) = 0.005. \quad (12)$$

Such a small effect will hardly be measurable at the LHC. Nevertheless, the fact that the SM value of  $A_{PV}$  is so small, makes this observable an ideal experimental sensor for tracing possible new parity-violating interactions in  $t\bar{t}$  production.

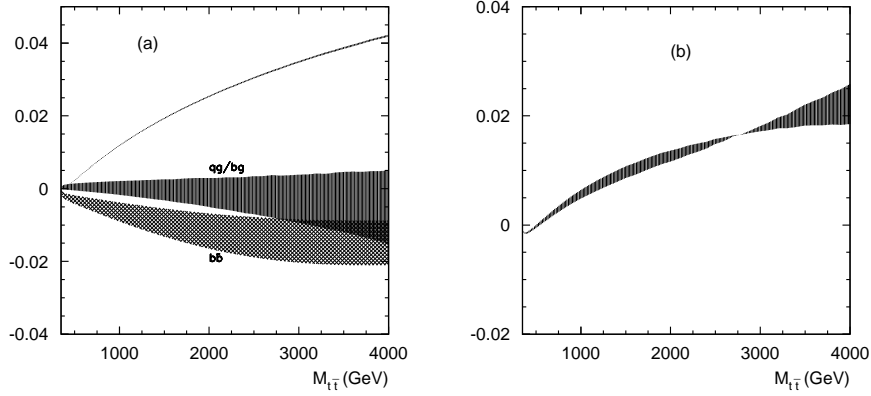


Figure 3: a) Contributions of the various partonic subprocesses to the helicity asymmetry  $\Delta_{hel}$ : initial states  $q\bar{q}$  ( $q \neq b$ ) and  $gg$  (thin line),  $qg$  and  $\bar{q}g$  ( $q = u, \dots, b$ ) (vertically hatched area), and  $b\bar{b}$  (cross hatched area). b) Sum of the three contributions shown in a) as computed in [11].

Thus  $A_{PV}$  should be known as precisely as possible within the SM. On the experimental side, it remains to be investigated with which precision can actually be measured by an LHC experiment. Given the results from simulations [23, 24] that asymmetries related to  $W$ -boson helicity fractions from top-quark decay should be measurable at the LHC with an overall uncertainty of  $\sim 1\%$ , it seems not unrealistic to expect that  $A_{PV}$ , which is of a similar type as these asymmetries, should eventually be measurable with a precision of  $\sim 1\%$ . One may also consider PV double spin-asymmetries. However these are equivalent, if CP invariance holds, to the corresponding single-spin asymmetries such as  $Z_{hel}$ , which are experimentally much more powerful. These and other spin issues are discussed in [10].

The Standard Model predictions of Fig. 3 and of (12) may be used as reference values in searches for parity-violating effects in hadronic  $t\bar{t}$  production and decay at the LHC. Which new physics effects could possibly lead to an asymmetry  $A_{PV}$  at the level of a few percent? Obvious candidates would be new heavy  $s$ -channel resonances that couple to  $t\bar{t}$  pairs strongly and in a parity-violating way. In two-Higgs doublet or supersymmetric models the radiative corrections to the  $t\bar{t}$  production amplitudes can lead to asymmetries larger than those given (12) if the new particles are not too heavy [16].

Finally, a word on how the weak corrections change in the presence of cuts.

If one takes into account only  $t\bar{t}$  events with  $p_T \geq p_{Tmin}$ , the corrections i), ii) will not change significantly, as long as  $p_{Tmin}$  is not too large. Choosing, for instance,  $p_{Tmin} = 30$  GeV does not lead to a significant change of the results shown in Figs. 1 - 3. Eventually, the weak corrections to the distributions discussed here should be evaluated in conjunction with the known NLO QCD corrections, for which the NLO PDF, in particular a NLO  $b$ -quark PDF, are to be used. The NLO  $b$ -quark PDF enhances the  $b$ -quark induced weak contribution to the  $M_{t\bar{t}}$  distribution and to  $\Delta_{hel}$  at large  $M_{t\bar{t}}$ .

To summarize: distributions and asymmetries are key observables in the detailed exploration of the dynamics of top quarks at the LHC, which should eventually be possible up to energy scales of a few TeV. Therefore these observables should be predicted within the SM as precisely as possible. For this reason, we have analyzed the electroweak corrections to hadronic  $t\bar{t}$  pair production, and computed the effect of these corrections on the top-quark transverse momentum distribution and on the  $t\bar{t}$  invariant mass distribution. For the LHC these corrections are not negligible with respect to the QCD corrections, especially at large  $p_T$  and  $M_{t\bar{t}}$ , respectively. Furthermore, we have computed a parity-violating forward-backward asymmetry  $A_{PV}$ , which is induced by the weak interaction. The fact that the SM value of  $A_{PV}$  is very small makes this observable an ideal tool to search for new PV interactions in  $t\bar{t}$  production.

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